Canonical Asymmetric Coupled-Resonator Filters

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Abstract—**A direct (noniterative) procedure is presented for realizing canonical, structurally asymmetric lowpass prototypes for coupled-resonator bandpass filters with "bridge" couplings. An asymmetric prototype is obtained from the canonical symmetric prototype (which is realizable without matrix methods) by applying simple plane rotations to the coupling matrix. The resulting asymmetric prototype may be a more desirable structure, and may have fewer couplings, than the canonical symmetric prototype. The procedure is applicable to filters with symmetric or asymmetric frequency responses.**

I. Introduction

THE USE OF nonsequential or "bridge" couplings in a narrow-bandpass coupled-resonator filter permits great flexibility in the choice of response characteristics. This is particularly advantageous when high selectivity and low passband distortion requirements are simultaneously imposed on a filter. In the first step of synthesis, an *approximation* in the form of a transducer function (or its inverse, the scattering parameter S_{21}) is found that is in accordance with both the filter specifications and the anticipated filter structure [2], [7]. In the second, or *realization*, step of synthesis, the element values of a specific filter circuit are obtained. The subject of this paper is the realization of canonical, structurally asymmetric low-pass prototype circuits for coupled-resonator filters with bridge couplings.

The realization procedure requires first that a canonical *symmetric* prototype network be realized [1]. This prototype can be realized without any matrix manipulations because the bisected even- and odd-mode networks of the symmetric structure have no bridge couplings. As a result, they are developed as simple ladder-like direct-coupled networks. The synthesis can also be carried out in a transformed frequency variable, which simplifies the approximation procedure and increases the overall numerical accuracy [2].

For a given application, the symmetric prototype may not be the optimum structure. In such instances a series of specific plane rotations are applied to the coupling matrix, transforming it into a more suitable asymmetric network. This realization procedure is a direct (noniterative) method, and does not require the formation of an initially noncanonical coupling matrix [3]. The procedure leads to some interesting canonical prototypes, including practical structures for filters of odd degree with symmetric frequency responses, and general filters with asymmetric frequency responses.

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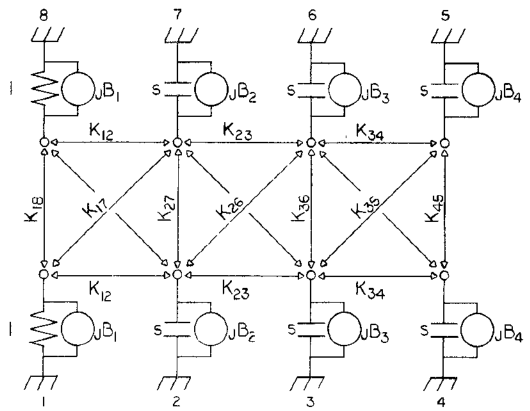

Fig. 1. $n = 6$ canonical symmetric prototype, asymmetric frequency response.

II. General Considerations

A. Circuit Representation

A nodal-admittance-matrix representation is used for lossless doubly terminated low-pass-prototype filters: $\boldsymbol{Y} = \boldsymbol{G} + s\boldsymbol{C} + j\boldsymbol{K}$, where $s = j\omega$. The degree of the filter is equal to n, the number of resonators. The order of $\boldsymbol{Y}$ is $n+2$, because the unit terminations, at nodes 1 and $n+2$, are separated by admittance inverters from the unit capacitors (resonators), which are connected to nodes 2 through $n+1$ (see Fig. 1). Thus the conductance matrix $\boldsymbol{G}$ has zero elements everywhere except for $G_{11} = G_{n+2} = 1$, and the capacitance matrix $\boldsymbol{C}$ is similar to the identity matrix except that $C_{11} = C_{n+2} = 0$.

The elements of the constant-susceptance coupling matrix $\boldsymbol{K}$ correspond to the transfer susceptances of admittance-inverter couplings between resonators (K_{ij}) and to node-shunting constant susceptances ($K_{ii} = B_i$). The matrix $\boldsymbol{K}$ is symmetric ($K_{ij} = K_{ji}$) since the filter is reciprocal. If the network is structurally symmetric, $\boldsymbol{K}$ will also be symmetric about the cross-diagonal: $K_{ij} = K_{k-j,k-i}$, where $k = n+3$.

B. Canonical Property

In addition to structural symmetry or asymmetry, coupled-resonator prototypes can be classified according to the symmetry or asymmetry of the frequency response. The

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response is symmetric if, in the low-pass frequency domain, the loss $\alpha(\omega)$ is an even function of frequency, and the phase $\beta(\omega)$ is an odd function. A prototype is considered to be *canonical* only if the number of couplings in the network is the minimum number required for the given structural and frequency-response symmetry characteristics. For *even-n* and *symmetric* frequency responses, symmetric and asymmetric canonical structures have the same number of couplings. In all other instances, however, the symmetric prototype can be transformed into a canonical asymmetric structure with fewer couplings.

C. Prototype Transformations

In order to preserve the transducer response of the network, transformations of $\boldsymbol{Y}$ that involve the termination nodes must be excluded. This implies that for a rotation in the (i, j)-plane neither i nor j can equal 1 or $n+2$. With this restriction, both $\boldsymbol{G}$ and $\boldsymbol{C}$ are invariant under orthogonal transformations, and therefore an allowed plane rotation $\boldsymbol{S}^T\boldsymbol{Y}\boldsymbol{S}$ will affect only the coupling matrix $\boldsymbol{K}$. In general, the structural symmetry of a network will be destroyed by a plane rotation.

The transformation procedure is based on Givens' method for solution of the algebraic eigenvalue problem [4], in which a sequence of plane rotations are used to reduce a real, symmetric matrix to tridiagonal form. By applying only certain such rotations to the coupling matrix, particular bridge couplings can be eliminated from the prototype network. Equations for the elements of a matrix after a plane rotation are given in Appendix A, where it is shown that in order to transform a coupling element value K_{ij} to zero, a rotation in the $(i+1, j)$-plane is made through the angle $\phi = \tan^{-1}(K_{ij}/K_{i,i+1})$. This transformation may or may not create another coupling elsewhere in the network, as will subsequently be shown.

III. Asymmetric-Response Filters

The more general type of filter with an asymmetric frequency response will be treated first. The transformation procedure will be demonstrated for an $n=6$ filter, whose structurally symmetric prototype [1] is shown in Fig. 1. The coupling matrix is

$$\boldsymbol{K} = \begin{bmatrix} B_1 & K_{12} & 0 & 0 & 0 & 0 & K_{17} & K_{18} \\ & B_2 & K_{23} & 0 & 0 & K_{26} & K_{27} & K_{17} \\ & & B_3 & K_{34} & K_{35} & K_{36} & K_{26} & 0 \\ & & & B_4 & K_{45} & K_{35} & 0 & 0 \\ & & & & B_4 & K_{34} & 0 & 0 \\ & & & & & B_3 & K_{23} & 0 \\ & & & & & & B_2 & K_{12} \\ & & & & & & & B_1 \end{bmatrix}$$

where only the upper part of the matrix is shown since it is symmetric. The structural symmetry of the network is evident by the symmetry of the matrix about the cross diagonal, e.g., $K_{46} = K_{35}$, etc. The number of bridge couplings is nine.

By performing the following sequence of rotations: (2, 7)-plane, $\phi = \tan^{-1}(K_{17}/K_{12})$; (3, 6)-plane, $\phi = \tan^{-1}(K_{26}/K_{23})$; and (4,5) = plane, $\phi = \tan^{-1}(K_{35}/K_{34})$, the couplings K_{17}, K_{26}, and K_{35} are eliminated without creating couplings elsewhere in the network. The resulting coupling matrix, with six remaining bridge couplings, is

$$\boldsymbol{K} = \begin{bmatrix} B_1 & K_{12} & 0 & 0 & 0 & 0 & 0 & K_{18} \\ & B_2 & K_{23} & 0 & 0 & 0 & K_{27} & K_{28} \\ & & B_3 & K_{34} & 0 & K_{36} & K_{37} & 0 \\ & & & B_4 & K_{45} & K_{46} & 0 & 0 \\ & & & & B_5 & K_{56} & 0 & 0 \\ & & & & & B_6 & K_{67} & 0 \\ & & & & & & B_7 & K_{78} \\ & & & & & & & B_1 \end{bmatrix}$$

where all nonzero elements except $K_{11} = K_{88} = B_1$, and K_{18} have been changed.

Further transformations can be applied to $\boldsymbol{K}$, but these will not reduce the number of couplings. For example, rotations with the appropriate angles in the sequence (3,7), (4,6), (4,7), (5,6), (5,7), and (6,7) will result in a coupling matrix

$$\boldsymbol{K} = \begin{bmatrix} B_1 & K_{12} & 0 & 0 & 0 & 0 & 0 & K_{18} \\ & B_2 & K_{23} & 0 & 0 & 0 & 0 & K_{28} \\ & & B_3 & K_{34} & 0 & 0 & 0 & K_{38} \\ & & & B_4 & K_{45} & 0 & 0 & K_{48} \\ & & & & B_5 & K_{56} & 0 & K_{58} \\ & & & & & B_6 & K_{67} & K_{68} \\ & & & & & & B_7 & K_{78} \\ & & & & & & & B_1 \end{bmatrix}.$$

The number of bridge couplings is still six, but they have all been shifted to the last column and row of the matrix.

Two distinct series of transformations have been described. In the first, the bridge couplings above the cross diagonal (K_{17}, K_{26}, and K_{35}) were eliminated, and each plane rotation reduced the number of couplings by one. Those elements could have been eliminated in any order. This series of transformations will result in a canonical asymmetric prototype of the form shown in Fig. 2(a).

In the second series of transformations, bridge-coupling elements on or below the cross diagonal (except those in the last column) were eliminated. The elimination of an element K_{ij}, however, created a new element $K_{i+1,j+1}$. Unless that element was in the last column, it also was eliminated. This series of transformations should proceed sequentially by rows, and by column within each row. Eventually all bridge couplings can be shifted into the last column and row of the matrix, resulting in the canonical asymmetric prototype of Fig. 2(b).

These transformation procedures and prototypes are applicable to networks of both even and odd degree. No more than $\mathrm{Int}(n/2)$ plane rotations are required to reduce the prototype to a canonical asymmetric form, which will have a maximum of n bridge couplings.

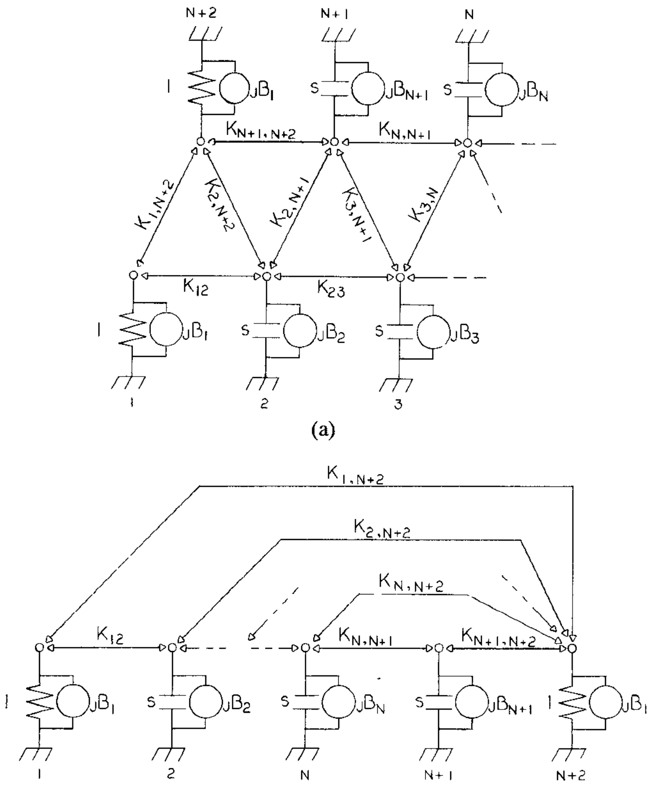

Fig. 2. Canonical asymmetric prototypes, asymmetric frequency response.

IV. Symmetric-Response Filters

The canonical symmetric prototype for a filter with a symmetric frequency response is characterized by a coupling matrix for which $K_{ij} = 0$ if $i + j$ is an *even* number. (The detailed proof of this is given in Appendix B.) As a result, the bridge couplings for even n appear only on the cross diagonal, while for odd n they appear only above and below the cross diagonal. Because of this difference, the application of the first series of transformations to the coupling matrix of even- and odd-degree filters will be discussed separately.

A. Filters of Even Degree

The canonical symmetric prototype for even n is shown in Fig. 3. For an $n = 6$ filter the coupling matrix is

$$K = \begin{bmatrix} 0 & K_{12} & 0 & 0 & 0 & 0 & 0 & K_{18} \\ & 0 & K_{23} & 0 & 0 & 0 & K_{27} & 0 \\ & & 0 & K_{34} & 0 & K_{36} & 0 & 0 \\ & & & 0 & K_{45} & 0 & 0 & 0 \\ & & & & 0 & K_{34} & 0 & 0 \\ & & & & & 0 & K_{23} & 0 \\ & & & & & & 0 & K_{12} \\ & & & & & & & 0 \end{bmatrix}. \tag{1}$$

It is clear that since there are no coupling elements above the cross diagonal, the first series of transformations cannot be applied to filters of even degree. Hence the number of couplings cannot be reduced, and the canonical symmetric and asymmetric structures have the same number of bridge couplings.

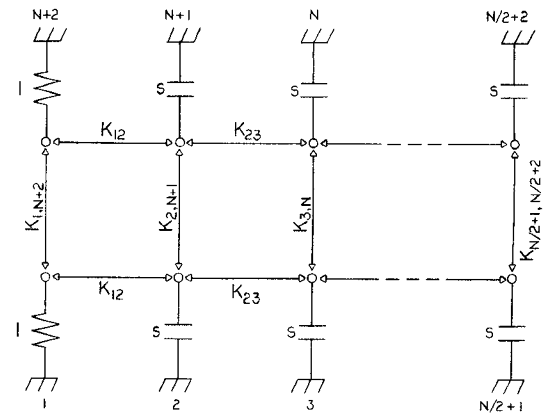

Fig. 3. Even-n canonical symmetric prototype, symmetric frequency response.

B. Filters of Odd Degree

The form of the canonical symmetric prototype for odd n is shown in Fig. 4(a). For an $n = 5$ filter,

$$K = \begin{bmatrix} 0 & K_{12} & 0 & 0 & 0 & K_{16} & 0 \\ & 0 & K_{23} & 0 & K_{25} & 0 & K_{16} \\ & & 0 & K_{34} & 0 & K_{25} & 0 \\ & & & 0 & K_{34} & 0 & 0 \\ & & & & 0 & K_{23} & 0 \\ & & & & & 0 & K_{12} \\ & & & & & & 0 \end{bmatrix}.$$

Applying the first series of transformations in the (2,6) and (3,5) planes, K_{16} and K_{25} are eliminated. The resulting coupling matrix is

$$K = \begin{bmatrix} 0 & K_{12} & 0 & 0 & 0 & 0 & 0 \\ & 0 & K_{23} & 0 & 0 & 0 & K_{27} \\ & & 0 & K_{34} & 0 & K_{36} & 0 \\ & & & 0 & K_{45} & 0 & 0 \\ & & & & 0 & K_{56} & 0 \\ & & & & & 0 & K_{67} \\ & & & & & & 0 \end{bmatrix}. \tag{2}$$

After the first series of transformations, the canonical asymmetric prototype has half the number of bridge couplings as that of the canonical symmetric prototype. Comparing the form of (2) with that of (1), the odd-degree prototype is similar to the symmetric structure of degree $n - 1$, with an extra node and resonator in the circuit. The resulting canonical asymmetric prototype is shown in Fig. 4(b).

C. Further Transformation

Although the first series of transformations, which reduces the number of couplings, is not applicable to filters

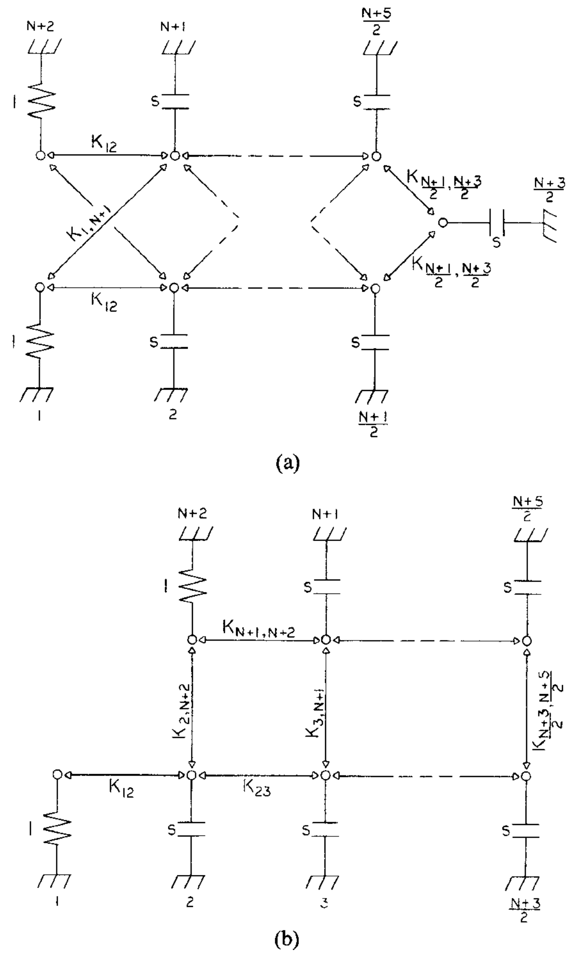

Fig. 4. Odd-n canonical prototypes. (a) Symmetric prototype. (b) Asymmetric prototype.

of even degree, additional transformations can be applied to obtain a more desirable structure for a particular application. An example of this is given in Section V-B, in which Pfitzenmaier's prototype for an asymmetric dual-mode filter [5] is obtained.

For filters of both even and odd degrees, the second series of transformations can be applied to shift all bridge couplings into the last column and row of the coupling matrix. This procedure will lead to canonical prototypes similar to that of Fig. 2(b), except that the bridge couplings $K_{i,n}$, where $i+n$ is even, as well as the resonator-shunting elements B_i, will be missing. This form of the prototype was described by Easter and Powell [6] for a filter of degree five. They were apparently the first to consider multiple couplings to a termination.

V. Design Examples

A. Filter of Degree Three

Easter and Powell [6] constructed a three-resonator waveguide filter based on a symmetric elliptic-function low-pass prototype response. The maximum passband SWR ripple was 1.15, and the stopband had a minimum loss of 30 dB with a loss pole on each side of the passband. For this example, the nonzero elements of the coupling matrix for the canonical symmetric prototype (Fig. 4(a)) are

$$K_{12} = K_{45} = 1.1524 \qquad K_{14} = K_{25} = -0.6251$$
$$K_{23} = K_{34} = 1.1209.$$

A single rotation in the (2,4)-plane results in a canonical asymmetric prototype (Fig. 4(b)) whose nonzero coupling elements are

$$K_{12} = 1.1541$$
$$K_{23} = 1.0586 \qquad K_{25} = -0.1248$$
$$K_{34} = 1.1800$$
$$K_{45} = 1.1473.$$

B. Filter of Degree Six

Pfitzenmaier [5] constructed a six-resonator square-waveguide dual-mode filter, for which the input and output resonators are located in adjacent cavities. He used a symmetric elliptic-function low-pass prototype response with a minimum passband return loss ripple of 26 dB, and a minimum stopband loss of 25 dB with two loss poles in

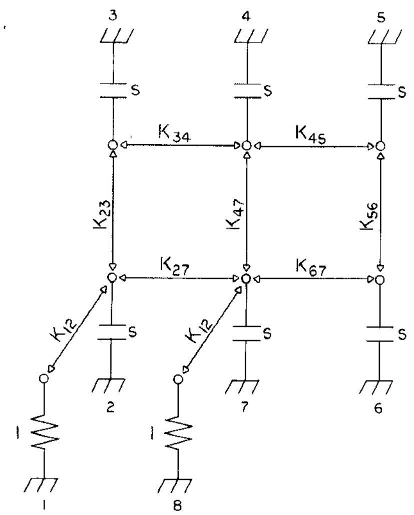

Fig. 5. Pfitzenmaier's $n = 6$ canonical asymmetric prototype for dual-mode filters.

each stopband. The nonzero coupling elements of the canonical symmetric prototype (Fig. 3) are

$$K_{12} = K_{78} = 1.1010$$
$$K_{23} = K_{67} = 0.9056 \qquad K_{27} = 0.1907$$
$$K_{34} = K_{56} = 0.5003 \qquad K_{36} = -0.4756$$
$$K_{45} = 0.8908.$$

By applying a (4,6)-plane rotation to make $K_{36} = 0$, the coupling K_{47} is created. The resulting canonical asymmetric prototype is shown in Fig. 5, and the nonzero couplings are

$$K_{12} = K_{78} = 1.1010$$
$$K_{23} = 0.9056 \qquad K_{27} = 0.1907$$
$$K_{34} = 0.6903$$
$$K_{45} = 0.3009 \qquad K_{47} = -0.6240$$
$$K_{56} = 0.9764$$
$$K_{67} = 0.6565.$$

VI. Conclusion

It has been shown that a minimal number of matrix manipulations are required to transform a canonical symmetric prototype into a canonical asymmetric prototype. Depending on the symmetry of the frequency response, no more than Int$(n/2)$ plane rotations are required to reduce the network to a canonical asymmetric form. Further transformations may then be applied to shift the locations of bridge couplings in the network, although no further reduction in the number of couplings is possible.

Appendix A

Plane Rotations

When a real, symmetric matrix A is transformed by a rotation of angle ϕ in the (p, q) plane, where it is assumed that $p < q$, the elements of the resulting matrix $\boldsymbol{B}$ are

$$B_{pp} = A_{pp}\cos^2\phi + A_{qq}\sin^2\phi + A_{pq}\sin 2\phi$$
$$B_{qq} = A_{pp}\sin^2\phi + A_{qq}\cos^2\phi - A_{pq}\sin 2\phi$$
$$B_{pq} = B_{qp} = \tfrac{1}{2}(A_{qq} - A_{pp})\sin 2\phi + A_{pq}\cos 2\phi$$
$$\left.\begin{aligned} B_{ip} &= B_{pi} = A_{ip}\cos\phi + A_{iq}\sin\phi \\ B_{iq} &= B_{qi} = -A_{ip}\sin\phi + A_{iq}\cos\phi \end{aligned}\right\}, \qquad i \neq p \text{ or } q$$
$$B_{ij} = A_{ij}, \qquad i \text{ and } j \neq p \text{ or } q.$$

In Givens' method [4], a rotation in the $(i+1, j)$ plane is performed to make $B_{ij} = -A_{i,i+1}\sin\phi + A_{ij}\cos\phi = 0$. The angle of rotation is therefore $\phi = \tan^{-1}(A_{ij}/A_{i,i+1})$.

Appendix B

Symmetric Frequency Response, Symmetric Prototype

The purpose here is to show that for the canonical symmetric low-pass prototype [1], a symmetric frequency response will result if all elements K_{ij} of the coupling matrix are zero, where $i + j$ is an *even* number.

A. Requirements for Symmetric Response

Let B_+ and B_- be the even- and odd-mode input susceptances, respectively, for the unterminated lossless symmetric prototype. (These are identical to the susceptances of the arms of an equivalent symmetric lattice network.) The transducer function for the filter with unit terminations is [7]

$$H = \frac{1}{S_{21}} = \frac{(1 + jB_+)(1 + jB_-)}{j(B_- - B_+)}.$$

The loss (α) in nepers and phase (β) in radians is given by $\alpha + j\beta = \ln H$. A symmetric frequency response implies that $\alpha(\omega) = \alpha(-\omega)$ and $\beta(\omega) = -\beta(-\omega)$. The requirement that β is an odd function of ω implies that

$$\frac{\mathrm{Im}(H)}{\mathrm{Re}(H)} = \frac{B_+B_- - 1}{B_+ + B_-} = \text{odd function of } \omega.$$

Thus, B_+B_- must be even, $B_+ + B_-$ must be odd, and therefore

$$\mathrm{Ev}(B_+)\cdot\mathrm{Od}(B_-) + \mathrm{Ev}(B_-)\cdot\mathrm{Od}(B_+) = 0$$
$$\mathrm{Ev}(B_+) = -\mathrm{Ev}(B_-).$$

These relationships can be satisfied in one of two ways:

$$\left.\begin{aligned} \mathrm{Ev}(B_+) &= -\mathrm{Ev}(B_-) \neq 0 \\ \mathrm{Od}(B_+) &= \mathrm{Od}(B_-) \end{aligned}\right\} \qquad \text{(B1)}$$

or

$$\mathrm{Ev}(B_+) = \mathrm{Ev}(B_-) = 0. \qquad \text{(B2)}$$

It can be easily verified that if either (B1) or (B2) is satisfied, $|H|$, and therefore $\alpha(\omega)$, are even functions of frequency.

B. Even- and Odd-Mode Susceptances

Partial-fraction expansions of $B\pm$ for the prototype [1] will be examined, in which $K_{ij}=0$ if $i+j$ is an even number.

Even n: The appropriate continued-fraction expansions are

$$B\pm(\omega) = \pm K_{1,n+2} - \cfrac{K_{12}^2}{\omega \pm K_{2,n+1} - \cfrac{K_{23}^2}{\omega \pm \cdots}}$$

$$B\pm(-\omega) = \pm K_{1,n+2} - \cfrac{K_{12}^2}{-\omega \pm K_{2,n+1} - \cfrac{K_{23}^2}{-\omega \pm \cdots}}.$$

It is apparent that $B\pm(-\omega) = -B\mp(\omega)$. Examining this relationship in more detail reveals that $\mathrm{Ev}(B_+) = -\mathrm{Ev}(B_-)$ and $\mathrm{Od}(B_+) = \mathrm{Od}(B_-)$, thus satisfying (B1).

Odd n: The appropriate continued-fraction expansions are

$$B\pm(\omega) = -\cfrac{(K_{12}\pm K_{1,n+1})^2}{\omega - \cfrac{(K_{23}\pm K_{2,n})^2}{\omega - \cdots}}$$

$$B\pm(-\omega) = -\cfrac{(K_{12}\pm K_{1,n+1})^2}{-\omega - \cfrac{(K_{23}\pm K_{2,n})^2}{-\omega - \cdots}}.$$

In this instance, $B\pm(-\omega) = -B\pm(\omega)$, hence $\mathrm{Ev}(B\pm) = 0$ and equation (B2) is satisfied.

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